Problem 1 (3 parts, 25 points) **Encoders and Decoders**

Part A (6 points) Consider a priority encoder with the following behavior:

In_2	In_1	In_0	Valid	Out_1	Out_0
0	0	0	0	x	x
0	0	1	1	0	0
0	1	0	1	0	1
0	1	1	1	0	1
1	0	0	1	1	0
1	0	1	1	0	0
1	1	0	1	0	1
1	1	1	1	0	1

List the inputs (In_0, In_1, and In_2) in decreasing priority.

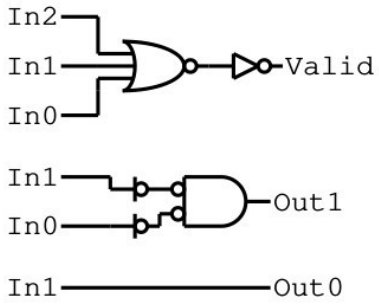
In_1 (highest priority) > In_0 (2nd highest priority) > In_2 (lowest priority)

Part B (12 points) Implement the priority encoder from part A using 2-input or 3-input NORs and inverters only.

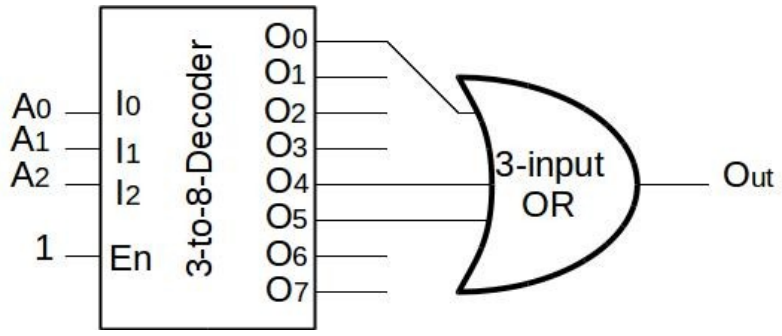
$Valid = In_0 + In_1 + In_2$

$Out_1 = \overline{In_0}\,\overline{In_1}$

$Out_0 = In_1$



Part C (7 points) Complete the circuit below to implement Out, whose behavior is shown in the truth table. Use only the decoder and one basic gate (e.g., AND, NAND, OR, NOR).



A0	A1	A2	Out
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Problem 2 (3 parts, 30 points) **Number Systems**

Part A (12 points) Convert the following notations:

binary notation	**decimal notation**
1101.011	**13.375**
101 1111.1	95.5
0111 1110 1000	**2024**
hexadecimal notation	**octal notation**
0x440	**2100**
17.68	27.32
0x178	**570**

Part B (12 points) For the 24 bit representations below, determine the most positive value and the step size (difference between sequential values). **Express all answers in decimal notation – do not leave your answer as 2 raised to an exponent** (e.g., say 4K, not 2^{12}). Fractions (e.g., 3/16ths) may be used. Signed representations are two's complement.

representation	most positive value	step size
unsigned integer (24 bits) . (0 bits)	**16M**	**1**
signed fixed-point (18 bits) . (6 bits)	**128K**	**1/64**
signed integer (24 bits) . (0 bits)	**8M**	**1**
signed fixed-point (20 bits) . (4 bits)	**512K**	**1/16**

Part C (6 points) What is the minimum number of bits needed to represent the following numbers in signed two's complement and as unsigned numbers?

Number:	**Min # bits for signed representation:**	**Min # bits for unsigned representation:**
-64	**7**	N/A
1204	**12**	**11**
64	**8**	**7**

Problem 3 (3 parts, 30 points) **Computation**

Part A (16 points) For each problem below, compute the operations using the rules of arithmetic, and indicate whether an overflow occurs assuming all numbers are expressed using a **five bit unsigned** and **five bit two's complement** representations.

	10011 + 11001	111 + 1010	1100 - 111	10001 - 10011
result	**01100**	**10001**	**00101**	**11110**
unsigned error?	**Yes**	**No**	**No**	**Yes**
signed error?	**Yes**	**Yes**	**No**	**No**

Part B (8 points) For each bit string below, what is the decimal number it represents if it uses a 5-bit unsigned representation and if it uses a 5-bit two's complement representation?

Bit string	Decimal (if unsigned representation)	Decimal (if 2's complement signed representation)
10110	**22**	**-10**
101.11	**5.75**	**-2.25**

Part C (6 points) A 26 bit floating point representation has a 16 bit mantissa field, a 10 bit exponent field, and one sign bit.

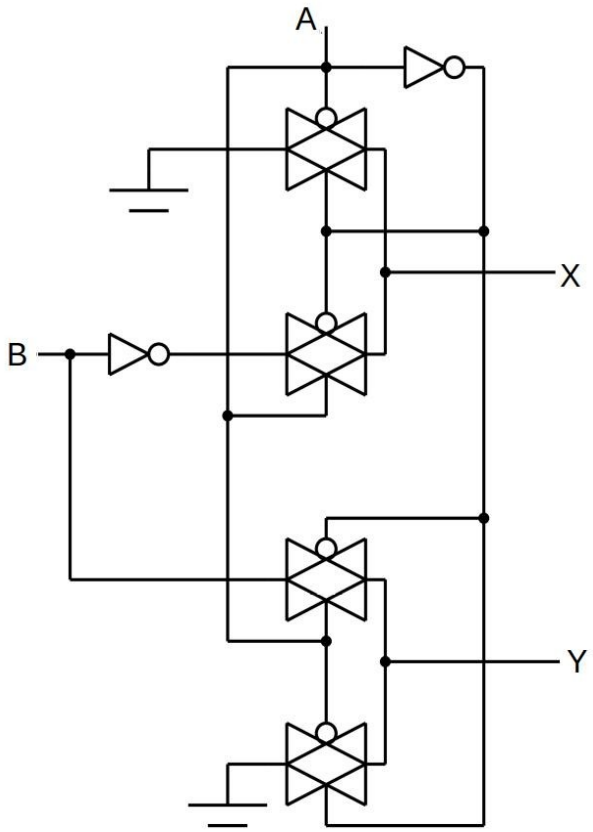
What is the largest value that can be represented (closest to infinity)? 2^{511}

What is the smallest value that can be represented (closest to zero)? 2^{-512}

How many decimal significant figures are supported? 5

Problem 4 (2 parts, 15 points) **Building Blocks and Pass Gates**

Consider the following circuit.



A	B	X	Y
0	0	**0**	**0**
0	1	**0**	**0**
1	0	**1**	**0**
1	1	**0**	**1**

Part A (8 points) Fill in the truth table to the right with its behavior.

Part B (7 points) What building block does this circuit implement? Express your answer in the form of n-to-m <type of building block> (e.g, 16-to-1 mux).

1-to-2 decoder